Chapter 3

Spacetime Symmetries

3.1 Introduction

Before discussing the forces of nature, we must understand the symmetries of space and time. These spacetime symmetries are more fundamental than gauge symmetries because they describe the arena in which all physical processes occur.

3.2 Translational Symmetry

Translational symmetry means that the laws of physics are identical at every location in space. There is no preferred place in the universe. By Noether's theorem, this symmetry implies conservation of linear momentum.

3.3 Time Translation Symmetry

The laws of physics are also independent of time. An experiment performed today gives the same result tomorrow if the conditions are unchanged. This symmetry gives rise to conservation of energy.

3.4 Rotational Symmetry

Rotational symmetry states that the laws of physics are unchanged when an experiment is rotated. No direction in empty space is fundamentally preferred. This symmetry leads to conservation of angular momentum.

3.5 Galilean Relativity

In classical mechanics, the laws of physics are invariant under Galilean transformations between observers moving at constant relative velocity. This approximation works well at speeds much smaller than the speed of light.

3.6 Lorentz Symmetry

Special relativity replaces Galilean symmetry with Lorentz symmetry. Space and time become intertwined into spacetime, and the speed of light is invariant for all inertial observers. Lorentz transformations mix space and time coordinates.

3.7 The Poincare Group

The complete symmetry of special relativity is the Poincare group, which combines spacetime translations with Lorentz transformations. Quantum field theory is constructed to respect this symmetry.

3.8 General Covariance

Einstein's General Theory of Relativity extends spacetime symmetry further. The laws of physics are invariant under arbitrary smooth coordinate transformations, a property known as general covariance or diffeomorphism invariance.

3.9 Symmetry and Conservation Laws

Noether's theorem connects spacetime symmetries with conserved quantities: time symmetry gives energy conservation, spatial symmetry gives momentum conservation, and rotational symmetry gives angular momentum conservation.

3.10 Spacetime Symmetries and the Standard Model

The Standard Model is built upon both spacetime and internal gauge symmetries. Every elementary particle transforms under representations of the Poincare group as well as the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$.

Chapter Summary

Spacetime symmetries provide the geometric foundation of modern physics. Translation, rotation, Lorentz symmetry, and the Poincare group define how particles and fields behave in space and time. Together with gauge symmetries, they form the mathematical framework of the Standard Model and quantum field theory.