

# Epicycles, Ellipses, & Apparent Motion: Ptolemaic, Copernican, Keplerian Models

## This work was motivated by viewing *Majestic Clockwork: How we Discovered our Solar System* \*

This simulation of classical models of planetary motion was constructed by comparing geocentric epicycle representations and heliocentric Copernican and Keplerian orbits of Venus, Mercury, and Jupiter. Using simple trigonometric formulation, **the apparent motion of each planet as seen from Earth** is built up from combinations of uniform circular motions, following both the Copernican kinematic construction and the historical Ptolemaic epicycle framework. For Venus, these models naturally reproduce retrograde motion with relatively simple geometry, while for Mercury they reveal increasing complexity, requiring additional epicycles to approximate observed behavior. See Plots Below: Ptolemaic Mercury Multi Epicycle. This program then contrasts these geometrically elaborate constructions with Kepler's elliptical orbits, **showing how a single heliocentric ellipse replaces multiple epicycles** while yielding the same apparent geocentric phenomena. Together, the models illustrate how retrograde motion arises as a projection effect of relative motion, and **why Mercury** played a central role in exposing the limitations of the epicycle system.

***Mathematically, Every Copernican Heliocentric Orbit, when transformed to Earth-centered coordinates, becomes an epicyclic geocentric curve. These are conceptual, not physical orbits.***

### Conceptual difference

*\*Majestic Clockwork Lecture*  
December 29, 2025

### Ptolemy and Copernicus both used circles, but they used them in fundamentally different ways.

In the Ptolemaic model, **Earth is fixed at the center.** Each planet moves on an epicycle, **whose center itself moves on a larger circle called the deferent.** direction" is a geocentric projection effect even inside Ptolemaic kinematics. "Retrograde occurs when the epicycle motion temporarily dominates the deferent motion in the geocentric longitude."

In the Copernican model, **planets move around the Sun, not Earth.** Retrograde motion is not real motion at all, but an apparent effect that arises when Earth's heliocentric motion is subtracted from another planet's heliocentric motion

***Mathematically, Ptolemy adds circles; Copernicus subtracts orbits.***

Ancient Geometric Astronomical Systems can *describe* motion. Only post-Newtonian Physics can *explain* motion.

## The difference in equations

### A. Ptolemaic-Style single-epicycle model (Earth fixed)

Earth is at the origin and never moves.

- $R_0$ : deferent radius  $R$
- $R_1$ : epicycle radius
- $\omega_D$ : deferent angular speed
- $\omega_e$ : epicycle angular speed
  - ✓ Retrograde motion comes from epicycle geometry.
  - ✓ Earth never moves

### Deferent angular speed ( $\omega_D$ )

is the constant angular rate at which the center of a planet's epicycle moves around Earth along the deferent circle in a Ptolemaic (P) geocentric model.

$$\begin{aligned}x_P(t) &= R_0 \cos(\omega_D t) + R_1 \cos(\omega_e t) \\y_P(t) &= R_0 \sin(\omega_D t) + R_1 \sin(\omega_e t)\end{aligned}$$

### Copernican model (heliocentric subtraction)

Earth and Venus both orbit the Sun.

Copernican (C) Heliocentric orbits:

$\omega_E$ : Earth heliocentric speed

$\omega_M$ : Mercury heliocentric speed

Apparent geocentric motion

- ✓ Retrograde motion arises from relative motion
- ✓ No physical epicycles exist

$$\begin{aligned}x_C(t) &= x_V(t) - x_E(t) \\y_C(t) &= y_V(t) - y_E(t)\end{aligned}$$

In the Copernican circular model, all planetary orbits share the same mathematical form; differences in apparent motion arise solely from orbital radius and period, not from different equations.

### Circular Motion of Earth and Venus

$$\begin{aligned}x_E(t) &= a_E \cos(\omega_E t), & y_E(t) &= a_E \sin(\omega_E t) \\x_V(t) &= a_V \cos(\omega_V t), & y_V(t) &= a_V \sin(\omega_V t)\end{aligned}$$

### In the Ptolemaic (P) System (Circular Functions) 150 AD

Retrograde motion is produced by epicycles added to a fixed Earth, whereas

### In the Copernican (C) System 1543 AD

the same phenomenon emerges naturally from subtracting Earth's heliocentric motion from that of another planet.

### In the Kepler (K) System (Elliptical Functions) 1619 AD

The orbits are plotted with ellipses.

January 3, 2026

# Ptolemaic, Copernican, and Kepler Models — Venus & Mercury

*Except where noted, this work was done using the Mathcad Programming Language.*

**Clock ~ 3 Earth Years:**  $t_{max} = 1100$  Days       $t_{max} := 1100$        $N_{\text{rev}} := 2960$        $n := 0, 1..2000$        $t := 0, \frac{t_{max}}{N} .. t_{max}$

$T_E := 365.256$        $T_V := 224.701$        $T_M := 87.969$        $X_E(t) := 0$        $Y_E(t) := 0$

## I. PTOLEMAIC (TRUE GEOCENTRIC)

Earth fixed. Retrograde from epicycles

### Venus (single epicycle), Ptolemaic-style

$$\omega_{DV} := \frac{2 \cdot \pi}{T_E} \quad \omega_{EV} := \frac{2 \cdot \pi}{T_V} \quad R_{0V} := 1 \quad R_{1V} := 0.72 \quad \text{Distance Units are in Scaled AU}$$

$$x_{PV}(t) := R_{0V} \cdot \cos(\omega_{DV} \cdot t) + R_{1V} \cdot \cos(\omega_{EV} \cdot t) \quad y_{PV}(t) := R_{0V} \cdot \sin(\omega_{DV} \cdot t) + R_{1V} \cdot \sin(\omega_{EV} \cdot t)$$

### Ptolemaic Multi-epicycle Model of Mercury's

irregular retrograde behavior played a decisive historical role by revealing the practical limits of epicycles and motivating Kepler's elliptical reformulation.

## Mercury (multi-epicycle, Ptolemaic-style) - Illustrative, not Ptolemy's exact parameters

$$\omega_{DM} := \frac{2 \cdot \pi}{T_E} \quad \omega_{E1} := \frac{2 \cdot \pi}{T_M} \quad \omega_{E2} := 2.6 \cdot \omega_{E1} \quad R_{0M} := 1 \quad R_{1M} := 0.38 \quad R_{2M} := 0.12$$

$$x_{PM}(t) := R_{0M} \cdot \cos(\omega_{DM} \cdot t) + R_{1M} \cdot \cos(\omega_{E1} \cdot t) + R_{2M} \cdot \cos(\omega_{E2} \cdot t)$$

$$y_{PM}(t) := R_{0M} \cdot \sin(\omega_{DM} \cdot t) + R_{1M} \cdot \sin(\omega_{E1} \cdot t) + R_{2M} \cdot \sin(\omega_{E2} \cdot t)$$

## II. HELIOCENTRIC COPERNICAN (CIRCULAR, RELATIVE MOTION)

$$a_E := 1 \quad a_V := 0.723 \quad a_M := 0.387 \quad \omega_E := \frac{2 \cdot \pi}{T_E} \quad \omega_V := \frac{2 \cdot \pi}{T_V} \quad \omega_M := \frac{2 \cdot \pi}{T_M}$$

$$x_E(t) := a_E \cdot \cos(\omega_E \cdot t) \quad y_E(t) := a_E \cdot \sin(\omega_E \cdot t)$$

$$x_V(t) := a_V \cdot \cos(\omega_V \cdot t) \quad y_V(t) := a_V \cdot \sin(\omega_V \cdot t)$$

$$x_M(t) := a_M \cdot \cos(\omega_M \cdot t) \quad y_M(t) := a_M \cdot \sin(\omega_M \cdot t)$$

$$x_{MC}(t) := x_M(t) - x_E(t) \quad y_{MC}(t) := y_M(t) - y_E(t)$$

$$x_{VC}(t) := x_V(t) - x_E(t) \quad y_{VC}(t) := y_V(t) - y_E(t)$$

## III. KEPLERIAN (ELLIPTICAL)

$$e_E := 0.0167 \quad e_V := 0.0068 \quad e_M := 0.2056 \quad ME(t) := \frac{2 \cdot \pi \cdot t}{T_E} \quad MV(t) := \frac{2 \cdot \pi \cdot t}{T_V} \quad MM(t) := \frac{2 \cdot \pi \cdot t}{T_M}$$

$$EE(t) := ME(t) + e_E \cdot \sin(ME(t)) \quad EV(t) := MV(t) + e_V \cdot \sin(MV(t)) \quad EM(t) := MM(t) + e_M \cdot \sin(MM(t))$$

$$b_E := a_E \cdot \sqrt{1 - e_E^2} \quad b_V := a_V \cdot \sqrt{1 - e_V^2} \quad b_M := a_M \cdot \sqrt{1 - e_M^2}$$

$$x_{EK}(t) := a_E \cdot (\cos(EE(t)) - e_E) \quad y_{EK}(t) := b_E \cdot \sin(EE(t))$$

$$x_{VK}(t) := a_V \cdot (\cos(EV(t)) - e_V) \quad y_{VK}(t) := b_V \cdot \sin(EV(t))$$

$$x_{MK}(t) := a_M \cdot (\cos(EM(t)) - e_M) \quad y_{MK}(t) := b_M \cdot \sin(EM(t))$$

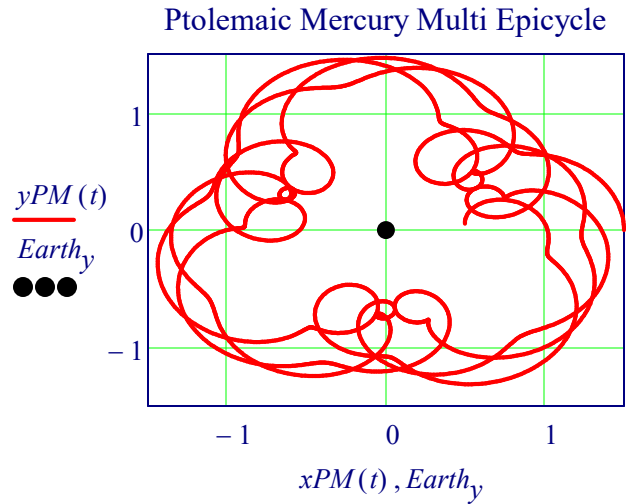
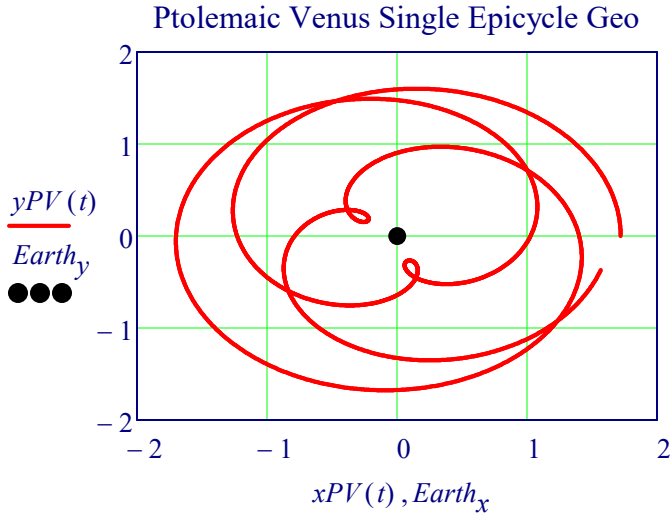
$$x_{VKg}(t) := x_{VK}(t) - x_{EK}(t) \quad y_{VKg}(t) := y_{VK}(t) - y_{EK}(t)$$

$$x_{MKg}(t) := x_{MK}(t) - x_{EK}(t) \quad y_{MKg}(t) := y_{MK}(t) - y_{EK}(t)$$

**Note:** Kepler solver is first-order; Mercury will have a small phase error.

# Orbital Simulations for Ptolemaic, Copernian, and Keplerian Models for Venus and Mercury

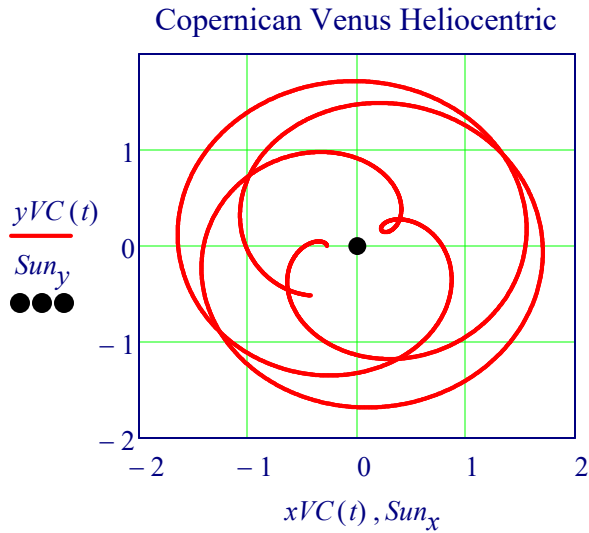
Time span shown:  $t = 0 \dots t_{max}$  (days). Apparent patterns repeat on synodic periods.



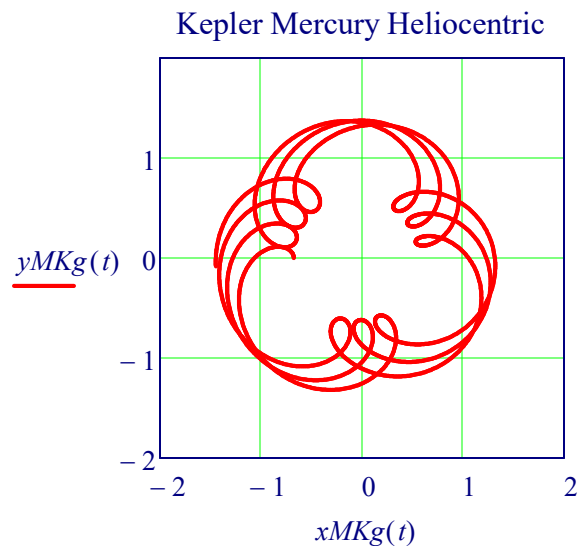
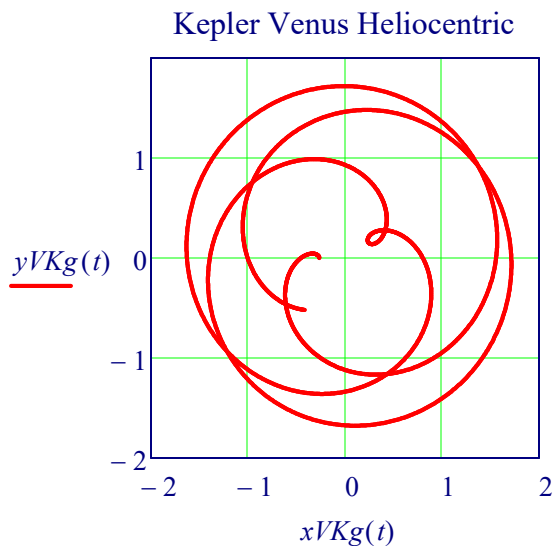
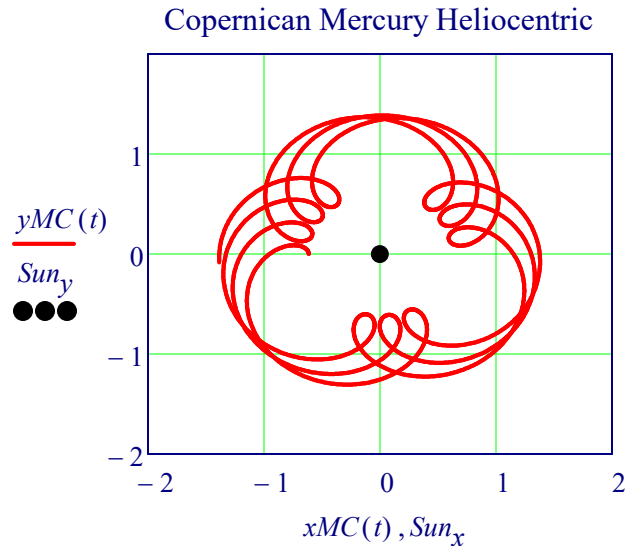
Copernican heliocentric orbit (inertial frame).

Heliocentric motion of a planet around the Sun, shown in an inertial frame. This curve represents the planet's physical trajectory in space and is not directly observable from Earth.

Copernican Venus — Geocentric Appearance (from heliocentric orbits)



Above: Multi-epicycle fit produces drifting loops (non-closure) without additional parameters.



## IV. DISTANCE-VS-TIME

$$r_{PV}(t) := \sqrt{x_{PV}(t)^2 + y_{PV}(t)^2}$$

$$r_{PM}(t) := \sqrt{x_{PM}(t)^2 + y_{PM}(t)^2}$$

$$r_{VC}(t) := \sqrt{x_{VC}(t)^2 + y_{VC}(t)^2}$$

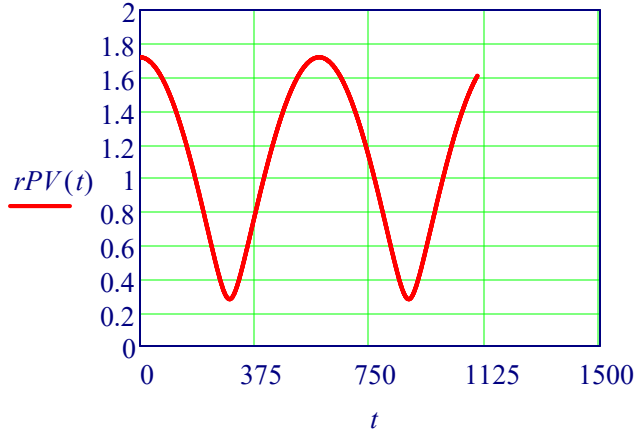
$$r_{MC}(t) := \sqrt{x_{MC}(t)^2 + y_{MC}(t)^2}$$

$$r_{VK}(t) := \sqrt{x_{VKg}(t)^2 + y_{VKg}(t)^2}$$

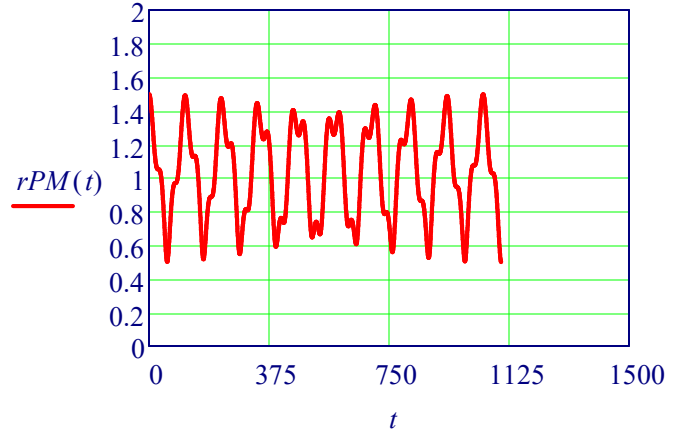
$$r_{MK}(t) := \sqrt{x_{MKg}(t)^2 + y_{MKg}(t)^2}$$

Although this **Ptolemaic Mercury Multi-Epicycle** construction reproduces retrograde loops, the curve does not close after one synodic period: successive retrograde loops drift in phase and shape, indicating that a finite epicycle fit cannot simultaneously match Mercury's motion over all times without adding further ad-hoc geometric correction.

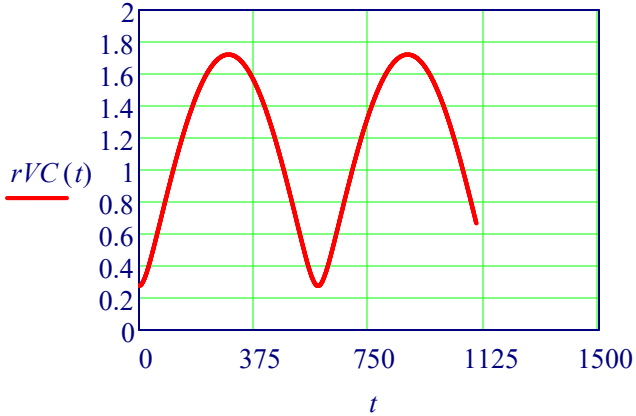
Ptolemaic Venus Synodic Cycle



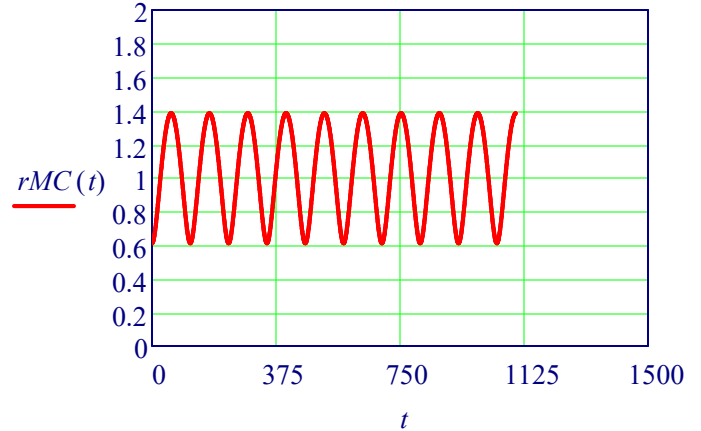
Ptolemaic Mercury Synodic Cycle



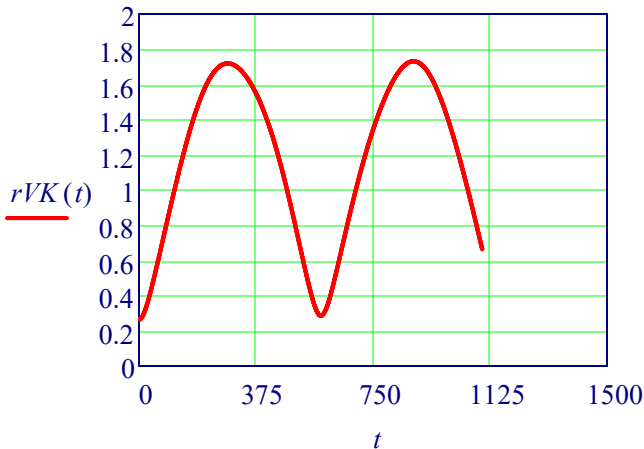
Copernicus Venus Synodic Cycle



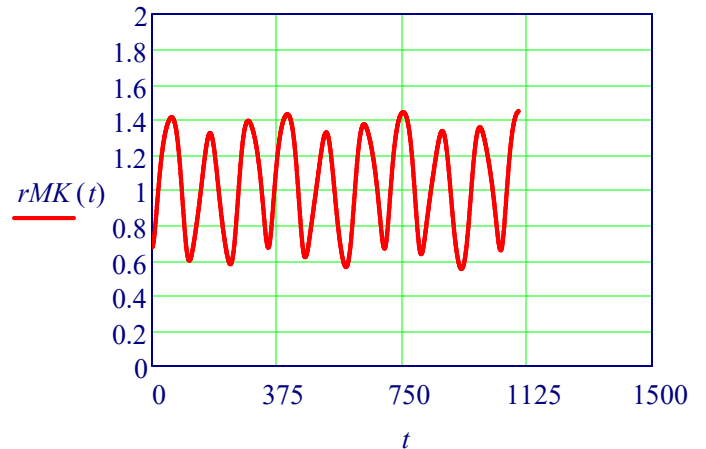
Copernicus Mercury Synodic Cycle



Kepler Venus Synodic Cycle



Kepler Mercury Synodic Cycle

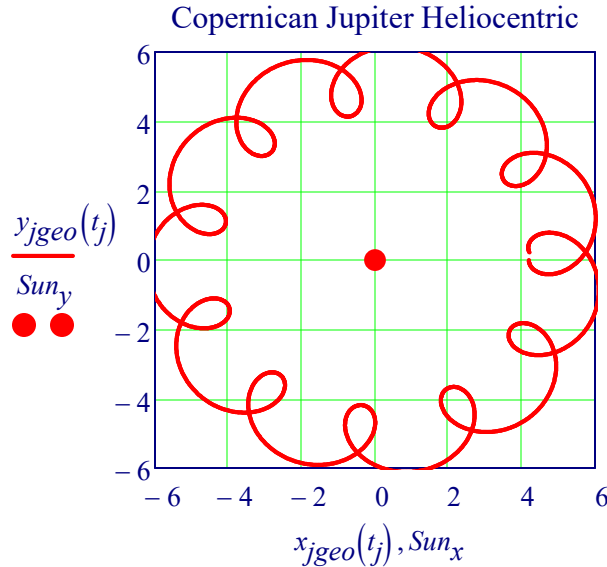


## Heliocentric Orbit for Jupiter

**Clock:  $t_{max} \sim 2$  Jupiter Years**      $t_{max} := 4400$       $a_J := 5.20$       $T_J := 4332.6$       $t_j := 0, \frac{t_{max}}{N} .. t_{max}$

$$x_J(t_j) := a_J \cdot \cos\left(2\pi \cdot \frac{t_j}{T_J}\right) \quad y_J(t_j) := a_J \cdot \sin\left(2\pi \cdot \frac{t_j}{T_J}\right)$$

$$x_{jgeo}(t_j) := x_J(t_j) - x_E(t_j) \quad y_{jgeo}(t_j) := y_J(t_j) - y_E(t_j)$$



### What is being plotted

This is not a physical orbit.  
It is Jupiter's apparent angular position against the fixed stars, with Earth held fixed at the center.

- So:
- Earth = center
  - Distance ignored
  - Direction on the sky matters

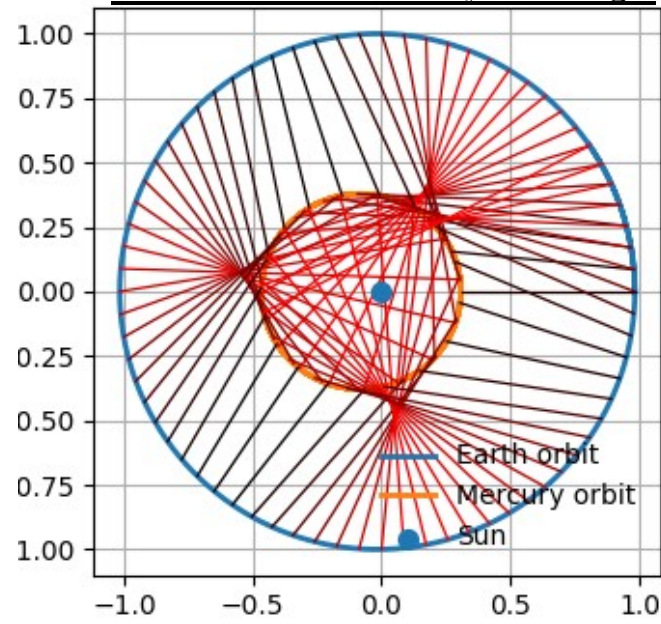
## What men saw looking at Mercury: Rays of Light Seen from Earth to Mercury

**Heliocentric Earth–Mercury lines of sight.** Each ray represents the instantaneous visual line from Earth to Mercury and is drawn only between the observer and the planet, with no extension beyond Mercury's orbital position. Heliocentric construction of the Earth–Mercury distance with the Sun fixed at the origin. Straight rays represent the instantaneous difference vector between Earth and Mercury. The apparent complexity of Mercury's motion arises entirely from relative motion, without the need for epicycles

Mathcad's XY plots struggle with many disconnected segments; rays shown below were generated in Python.

The outer blue ellipse is the orbit of the earth around the Mercury and the sun. The inner red ellipse is the orbit of Mercury. Heliocentric Earth–Mercury distance rays. Lines of sight are drawn from Earth to Mercury at uniform 5° intervals along Earth's orbit. (This produces uniform angular spacing and not steps in time.) This sampling better reflects how Mercury's apparent direction changes as Earth progresses around the Sun.

### View from Earth to Mercury Line-of-Sight



Distance Encoded Scheme  
Closest Mercury → black  
Farthest Mercury → red

Cause of Bunching Lines  
of sight lingers and pivots rapidly around a small region during the synodic cycle—so many consecutive time samples map to nearly the same direction/endpoint neighborhood.